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Robust Tracking Control Design for Spacecraft Under Control Input Saturation

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A continuous globally stable tracking control algorithm is presented for spacecraft in the presence of control input saturation, parametric uncertainty, and external disturbances. The proposed control algorithm has the following properties: 1) fast and accurate response in the presence of bounded disturbances and parametric uncertainty; 2) explicit accounting for control input saturation; and 3) computational simplicity and straightforward tuning. A detailed stability analysis of the resulting closed-loop system is included. It is shown that global stability of the overall system is guaranteed with continuous control even in the presence of bounded disturbances and parametric uncertainty. In the proposed controller a single parameter is adjusted dynamically in such a fashion that it is possible to prove that both attitude and angular velocity errors will tend to zero asymptotically. The stability proof is based on a Lyapunov analysis and the properties of the quaternion representation of spacecraft dynamics. One of the main features of the proposed design is that it establishes a straightforward relationship between the magnitudes of the available control inputs and those of the desired trajectories and disturbances even with continuous control. Numerical simulations are included to illustrate the spacecraft performance obtained using the proposed controller.

I. Introduction

AN important practical problem related to spacecraft control is that of control input saturation. This problem has attracted considerable interest in the existing literature. In a related work,[1] the authors use the Lyapunov approach to arrive at a saturated control design that was found to be effective in simulation studies. However, the stability analysis in the case of simultaneous stabilization of spacecraft attitude and angular velocity under control input saturation was not carried out. Wie and Lu[2] focus on the problem of rapid reorienting of a rigid spacecraft under sensor and actuator constraints. In the paper only the disturbance-free case is considered, and the knowledge of the spacecraft inertia matrix is needed to implement the controller. A similar problem was considered by Seywald[3] also in the disturbance-free case, and the control law again needs the information about the inertia matrix. Lo and Chen[4] designed a smooth sliding-mode attitude tracking controller. The design is nonglobal because it assumes that the fourth quaternion is nonzero for all time. In addition, well-estimated initial conditions are needed to implement the control law. Although sliding-mode control was used by Terui[5] to control both position and attitude of a spacecraft, control input saturation was not taken into account. In another related study,[6] the author designs a stabilizing controller for a spacecraft in a central gravitational field. The proposed dynamic controller achieves the objective of attitude stabilization under input saturation, inertia-matrix uncertainty, and external disturbance. However, the control law cannot be explicitly implemented, which was acknowledged by the author. Also, tracking of desired trajectories was not considered. In a recent paper,[7] the authors propose a dynamic tracking controller for the Euler–Lagrange systems. The knowledge of the system inertia matrix is needed to implement the controller. In addition, the asymptotic stability result is nonglobal. The authors considered the case when the velocities are not measurable. However, it is not clear if removing this constraint would remove the preceding drawbacks of the approach.

Globally stable control algorithms for stabilization of spacecraft attitude dynamics were reported recently.[8] The design of such algorithms is based on the variable structure approach to control systems design. These control algorithms have the following properties: 1) fast and accurate response in the presence of bounded external disturbances and parametric uncertainty; 2) explicit accounting for control input saturation; and 3) computational simplicity and straightforward tuning. An extension of the algorithms to the case of tracking has also been attempted.[9] However, the control algorithm that ensures the convergence of the tracking errors to zero is discontinuous, which results in chattering of the signals in the system. The use of an approximate sign function in the control law results in a system for which it could be shown that the tracking errors are bounded, but not that they converge to zero asymptotically. In addition, a Lyapunov-like function used in the stability analysis was not global in the sense of covering the entire state space of the system. In both papers, a dynamic controller was proposed where an adjustable parameter $k(t)$ was used to implement the controller. The major drawback of these results is that the adjustable parameter could only decrease or remain constant and, hence, could approach zero during the transient or asymptotically, which would lead to a situation in which it is not possible to conclude the convergence of the error quaternion to zero.

In this paper, based on the previous work, a new result is presented, loosely based on the previous results,[9] regarding the design of robust continuous tracking control algorithms for spacecraft. The algorithm is based on a continuous version of the variable structure control design approach and result in a globally stable overall system, as demonstrated using a suitable Lyapunov function. The proposed controller achieves asymptotic tracking and disturbance rejection with saturated inputs and is robust to parametric uncertainty in the spacecraft inertia matrix.

The paper is organized as follows: Preliminaries are given in Sec. II. Section III presents the case when the objective is to design a controller for tracking of angular velocities only in the presence of bounded disturbances and discusses the dynamic adjustment of a controller parameter to achieve disturbance rejection with continuous control. The extension of the results to the case of tracking of

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both angular velocities and spacecraft attitude is presented in Sec. IV. Section V contains simulation results, followed by conclusions and a list of references.

II. Preliminaries

A. Spacecraft Attitude Dynamics

The spacecraft is assumed to be a rigid body with actuators that provide torques about three mutually perpendicular axes. These axes define a body-fixed frame $\mathcal{B}$. The equations of motion are given by[10]

$$J\dot{\Omega} = -\Omega^{\times} J\Omega + u + z \tag{1}$$

$$\dot{\epsilon} = \tfrac{1}{2}\left(\epsilon^{\times} + \epsilon_0 I\right)\Omega \tag{2}$$

$$\dot{\epsilon}_0 = -\tfrac{1}{2}\epsilon^T\Omega \tag{3}$$

where $\Omega \in \mathbb{R}^3$ denotes the body angular velocity of the spacecraft with respect to an inertial frame $\mathcal{I}$, $J = J^T$ denotes the positive definite inertia matrix of the spacecraft, $\epsilon \in \mathbb{R}^3$ and $\epsilon_0 \in \mathbb{R}$ denote the Euler parameters that represent the orientation of $\mathcal{B}$ with respect to an inertial frame $\mathcal{I}$ and satisfy the constraint $\epsilon^T\epsilon + \epsilon_0^2 = 1$. In Eq. (2), I denotes a 3×3 identity matrix, whereas in Eq. (1) z denotes a vector of environmental disturbance torques. Further, $u \in \mathbb{R}^3$ denotes the control input vector, and $u \in \mathcal{S}_u = \{u : |u_i| \le u_m, i = 1, 2, 3\}$.

B. Desired Dynamics

In the case of tracking a desired rotational motion, the problem is formulated similarly as in the related work.[11] The desired motion of the spacecraft is specified by the attitude of a frame $\mathcal{D}$ whose orientation with respect to $\mathcal{I}$ is described by the Euler parameters $(\xi, \xi_0) \in \mathbb{R}^3 \times \mathbb{R}$ that satisfy the constraint $\xi^T\xi + \xi_0^2 = 1$. Let $\nu \in \mathbb{R}^3$ denote the angular velocity of $\mathcal{D}$ with respect to $\mathcal{I}$, which is equivalent to the desired angular velocity of the spacecraft expressed in the frame $\mathcal{D}$. The following assumption is made about ν and $\dot{\nu}$:

Assumption 1: There exist constants $\bar{\nu}_1 > 0$ and $\bar{\nu}_2 > 0$ such that $|\nu(t)| \le \bar{\nu}_1$ and $|\dot{\nu}(t)| \le \bar{\nu}_2$ for all $t \ge 0$.

Let $(\eta, \eta_0) \in \mathbb{R}^3 \times \mathbb{R}$ denote the Euler parameters representing the orientation of frame $\mathcal{B}$ with respect to $\mathcal{D}$. These parameters satisfy the constraint $\eta^T\eta + \eta_0^2 = 1$ and are related to (ξ, ξ_0) and (ϵ, ϵ_0) by the quaternion multiplication rule.[10] The corresponding rotation matrix $C = C(\eta, \eta_0) \in SO(3)$ is given by $C = (\eta_0^2 - \eta^T\eta)I + 2\eta\eta^T - 2\eta_0\eta^{\times}$. The angular velocity $\omega \in \mathbb{R}^3$ of $\mathcal{B}$ with respect to $\mathcal{D}$ is then $\omega = \Omega - C\nu$.

C. Error System

The tracking problem is solved if $\lim_{t\to\infty}\eta(t) = \lim_{t\to\infty}\omega(t) = 0$. This is equivalent to stabilization of ω and η, and the equations that govern their motion are given by

$$J\dot{\omega} = -(\omega + C\nu)^{\times} J(\omega + C\nu) + J(\omega^{\times} C\nu - C\dot{\nu}) + u + z \tag{4}$$

$$\dot{\eta} = \tfrac{1}{2}\left(\eta^{\times}\omega + \eta_0\omega\right) \tag{5}$$

$$\dot{\eta}_0 = -\tfrac{1}{2}\eta^T\omega \tag{6}$$

D. Desired Trajectory, Disturbances, and Control Inputs

For a given desired motion represented by $\nu(t)$ and $\dot{\nu}(t)$, define $\bar{\nu} \triangleq \sup_{t\ge 0}(|\nu(t)|^2 + |\dot{\nu}(t)|) = \bar{\nu}_1^2 + \bar{\nu}_2$. The constant $\bar{\nu}$ can be thought of as a measure of the speed and magnitude of the desired motion. It is assumed that $z(t) \in \mathcal{S}_z = \{z : |z| = |z_1| + |z_2| + |z_3| \le \bar{z},\ j = 1, 2, 3\}$ for all time, where the upper bound $\bar{z}$ on the magnitude of the disturbances is known. Let $\bar{\lambda}_J$ denote a known upper bound on the norm of J. The relationship between u_m and $\bar{z}$ is addressed in the following assumption.

Assumption 2: $u_m > \bar{\lambda}_J\bar{\nu} + \bar{z}$.

Loosely speaking, assumption 2 states that the available control authority is sufficient to simultaneously track the desired motion $\nu(t), \xi(t), \xi_0(t)$ and reject any disturbances from $\mathcal{S}_z$, which is a reasonable assumption in practice. The control objective is now stated as follows.

E. Control Objective

Design a control input $u(t) \in \mathcal{S}_u$ for the plant 1–3, whose tracking error dynamics is given by Eqs. (4–6), such that, for all physically realizable initial conditions, all $z \in \mathcal{S}_z$ and all $J = J^T > 0$ such that $\|J\| \le \bar{\lambda}_J$ and the following is achieved: $\lim_{t\to\infty}\eta(t) = \lim_{t\to\infty}\omega(t) = 0$.

III. Tracking of ω with Disturbance Rejection

In this section the disturbance rejection problem will be discussed first for the case when the control objective is to design a controller such that the spacecraft angular velocity tracks the desired motion while rejecting bounded time-varying disturbances. Based on the definition of the error $\omega = \Omega - C\nu$, the objective reduces to that of designing bounded continuous control inputs such that $\lim_{t\to\infty}\omega(t) = 0$ for all $z \in \mathcal{S}_z$ and all $\|J\| \le \bar{\lambda}_J$.

In this case the controller is proposed in the form

$$u_i = -u_m \cdot \omega_i \Big/ \left(|\omega_i| + k^2\delta\right), \qquad i = 1, 2, 3 \tag{7}$$

where $\delta > 0$. It is seen that the first term on the right-hand side contains a variable k that will be adjusted dynamically to ensure that $k(t) > 0$ for all time given $k(0) > 0$ and to ensure asymptotic disturbance rejection, as discussed next.

Let a Lyapunov function candidate be of the form

$$V(\omega, k) = \tfrac{1}{2}(\omega^T J\omega + k^2/\gamma) \tag{8}$$

Its first derivative along the solutions of Eq. (4) yields

$$\dot{V}(\omega, k) = -\omega^T H\omega - \omega^T g + \omega^T u + \omega^T z + \frac{k\dot{k}}{\gamma}$$

$$= -u_m \sum_{i=1}^{3} \frac{\omega_i^2}{|\omega_i| + k^2\delta} + \omega^T(z - g) + \frac{k\dot{k}}{\gamma} \tag{9}$$

where $H \triangleq (C\nu)^{\times}J + J(C\nu)^{\times}$ and $g \triangleq (C\nu)^{\times}J(C\nu) + JC\dot{\nu}$. It can be readily shown that $H = -H^T$ because J is symmetric and $(C\nu)^{\times}$ is skew symmetric. Therefore $\omega^T H\omega = 0$ for all $\omega \in \mathbb{R}^3$.

It is next noted that the following holds:

$$\frac{\omega_i^2}{|\omega_i| + k^2\delta} = |\omega_i| - \frac{|\omega_i|k^2\delta}{|\omega_i| + k^2\delta}$$

hence,

$$\dot{V}(\omega, k) \le -u_m \sum_{i=1}^{3}\left(|\omega_i| - \frac{|\omega_i|k^2\delta}{|\omega_i| + k^2\delta}\right) + |\omega|(\bar{z} + |g|) + \frac{k\dot{k}}{\gamma} \tag{10}$$

where $|\omega| = |\omega_1| + |\omega_2| + |\omega_3|$. The adjustment law for $k(t)$ is now chosen as

$$\dot{k} = -\gamma \sum_{i=1}^{3} \frac{u_m|\omega_i|k\delta}{|\omega_i| + k^2\delta} \tag{11}$$

This yields

$$\dot{V}(\omega, k) \le -(u_m - |g| - \bar{z})|\omega| \tag{12}$$

$$\le c|\omega| \le 0 \tag{13}$$

where $c > 0$ because, from assumption 2, $u_m > \bar{\lambda}_J\bar{\nu} + \bar{z}$, where $|g| \le \bar{\lambda}_J\bar{\nu}$, $\bar{\nu} \triangleq \sup_{t\ge 0}(|\nu(t)|^2 + |\dot{\nu}(t)|)$ and $\bar{\lambda}_J$ denotes a known upper bound on the norm of J.

It follows that ω and k are bounded. Let $u_m - \bar{\lambda}_J\bar{\nu} - \bar{z} = c$. By integrating $\dot{V}$ from 0 to ∞, one obtains

$$V(0) - V(\infty) \ge c\int_0^{\infty} |\omega(\tau)|\,\mathrm{d}\tau \tag{14}$$

Because the term on the left-hand side is bounded, it follows that $\omega \in \mathcal{L}^1$. Because u and ω are bounded, it follows that $\dot{\omega}$ is also bounded. Hence, from Barbalat's lemma[12] it follows that $\lim_{t\to\infty} \omega(t) = 0$.

For any given finite $\gamma > 0$, the integral

$$\int_0^\infty |\omega(\tau)|\,\mathrm{d}\tau$$

is bounded by some positive function $c_1(\gamma)$, that is,

$$c_1(\gamma) > \int_0^\infty |\omega(\tau)|\,\mathrm{d}\tau$$

A. Behavior of $k(t)$

It is seen that the potential problem with the algorithm (11) is that $k(t)$ could converge to zero before $\omega(t)$, and, therefore, cause chattering of the signals in the system because, for $k = 0$, the control laws become $u_i = -u_m \mathrm{sgn}(\omega_i)$. It will be shown next that the convergence rate of $k(t)$ can be adjusted to avoid its convergence to zero.

Assertion: For the adjustment law (11), for any given $k(0) = k_0 > 0$ and $\bar{k} > 0$ where $\bar{k} < k_0$, if there exists a $\gamma > 0$ such that

$$\gamma c_1(\gamma) \le \frac{k_0^2 - \bar{k}^2}{2u_m} \tag{15}$$

then $k(t) \ge \bar{k}$ for all $t \ge 0$.

Proof: It is first noted that the asymptotic convergence of $\omega(t)$ to zero is established independently of the behavior of $k(t)$ because $\dot{V}(\omega, k)$ from Eq. (12) and the subsequent arguments do not depend on k. Also, based on the discussion from the preceding paragraph $\omega(t)$ tends to zero asymptotically even if $k(t) = 0$.

Upon an examination of Eq. (11), $\lim_{t\to\infty} \omega(t) = 0$ implies that $\lim_{t\to\infty} \dot{k}(t) = 0$. Hence $k(t)$ will be moving slower and slower as the time increases. A potential problem is that $k(t)$ might go to zero before $\omega(t)$ has converged to zero, which can cause chattering of the control input. Therefore, the objective is to show that $k(t)$ will stay above some positive value $\bar{k}$ for all time.

From Eq. (11) because $u_m|\omega_i|k\delta/(|\omega_i| + k^2\delta) \le u_m k\delta$, the following inequality holds:

$$\dot{k} \ge -3\gamma u_m \delta k$$

Let $k(0) = k_0 > 0$. The preceding inequality can be integrated to obtain

$$k(t) \ge k_0 \exp(-3\gamma u_m \delta t) \tag{16}$$

This establishes a positive lower bound on $k(t)$ for all $t \in [0, \infty)$ that is independent of $\omega(t)$, and hence $k(t) \ge 0$ for all $t \in [0, \infty)$. In other words, $k(t)$ cannot cross zero or tend to zero in finite time, but only at infinity.

However, the objective here is to show an even stronger result, that is, derive conditions under which $k(t)$ is greater than some prespecified $\bar{k}$ for all time. This is discussed next.

Examining Eq. (11) again, it is noted that

$$\dot{k} \ge -\frac{u_m \gamma}{k} \sum_{i=1}^{3} |\omega_i|$$

which implies that

$$\dot{k} \ge -(u_m \gamma / k)|\omega| \tag{17}$$

Then integrating Eq. (17) from $t = 0$ to $t = \infty$ yields

$$\frac{k^2(\infty)}{2} - \frac{k_0^2}{2} \ge -u_m \gamma \int_0^\infty |\omega(\tau)|\,\mathrm{d}\tau$$
$$\ge -u_m \gamma c_1(\gamma)$$

If a $\gamma > 0$ exists such that

$$\gamma c_1(\gamma) \le \frac{k_0^2 - \bar{k}^2}{2u_m}$$

then $k(\infty) \ge \bar{k}$. Because $k(t)$ is a nonincreasing function of time, this implies that $k(t) \ge \bar{k}$ for all $t \ge 0$. ■

Assumption (15) imposes a condition that γ needs to satisfy in order to ensure that $k(t) \ge \bar{k}$ for all $t \ge 0$. However, it is seen from Eq. (15) that this condition depends on $c_1(\gamma)$, that is, there is an implicit condition that γ needs to satisfy. Finding an explicit condition for γ is the main technical difficulty associated with the proposed approach. However, in simulation studies, when γ is chosen to be small, that is, $\gamma \ll 1$, convergence of k to some nonzero value is always observed.

The extension of the preceding result to the case of tracking of both ω and η in the presence of bounded disturbances is discussed next.

IV. New Global Tracking Controller

In this paper the new global tracking controller for controlling both ω and η is proposed in the form

$$u_i = -\frac{u_m s_i}{|s_i| + k^2\delta}, \qquad i = 1, 2, 3 \tag{18}$$

where $\delta > 0$, and the coordinate transformation s is defined as

$$s = \omega + k^2\eta \tag{19}$$

The objective is to demonstrate that such a choice of the control law guarantees that the control objective is achieved. This is discussed next.

The following tentative Lyapunov function is chosen:

$$V(\omega, \eta, \eta_0, k) = \tfrac{1}{2}\left\{\omega^T J\omega + 2k^2\left[\eta^T\eta + (\eta_0 - 1)^2\right] + k^2/\gamma\right\} \tag{20}$$

Its derivative along the motions of the system yields

$$\dot{V}(\omega, \eta, \eta_0, k) = -\omega^T H\omega - \omega^T g + \omega^T u + \omega^T z + k\dot{k}\left[\frac{1}{\gamma} + 4(1 - \eta_0)\right] + k^2\eta^T\omega$$
$$= -u_m \sum_{i=1}^{3} \frac{\omega_i^2 + k^2\omega_i\eta_i}{|s_i| + k^2\delta} + \omega^T(-g + z) + k\dot{k}\left[\frac{1}{\gamma} + 4(1 - \eta_0)\right] + k^2\eta^T s - k^4\eta^T\eta$$

It is noted that the following holds:

$$\frac{\omega_i^2}{|s_i| + k^2\delta} = \frac{\omega_i^2}{\left|\omega_i + k^2\eta_i\right| + k^2\delta} \ge \frac{\omega_i^2}{|\omega_i| + k^2(1+\delta)}$$
$$= |\omega_i|\left(1 - \frac{k^2(1+\delta)}{|\omega_i| + k^2(1+\delta)}\right)$$

hence,

$$\dot{V}(\omega, \eta, \eta_0, k) \le -|\omega|(u_m - |g| - \bar{z}) + u_m \sum_{i=1}^{3}\left[\frac{|\omega_i|k^2(1+\delta)}{|\omega_i| + k^2(1+\delta)} - \frac{k^2\omega_i\eta_i}{|s_i| + k^2\delta}\right] + k\dot{k}\left[\frac{1}{\gamma} + 4(1 - \eta_0)\right] + k^2\eta^T s - k^4\eta^T\eta$$

The adjustment law for $k(t)$ is now chosen as

$$\dot{k} = \frac{\gamma k}{1 + 4\gamma(1 - \eta_0)}\left\{u_m \sum_{i=1}^{3}\left[\frac{\omega_i\eta_i}{|s_i| + k^2\delta} - \frac{|\omega_i|(1+\delta)}{|\omega_i| + k^2(1+\delta)}\right] - \eta^T s\right\} \tag{21}$$

which results in

$$\dot{V}(\omega, \eta, \eta_0, k) \le -|\omega|(u_m - |g| - \bar{z}) - k^4\eta^T\eta \tag{22}$$

Because $|g| \le \bar{\lambda}_J\bar{\nu}$, it follows that, under assumption 2, $\dot{V} \le 0$. This implies that ω and k are bounded. It is noted that η and η_0 are bounded by definition; hence, V is bounded.

Let $u_m - \bar{\lambda}_J\bar{\nu} - \bar{z} = c$. By integrating $\dot{V}$ from 0 to ∞, one obtains

$$V(0) - V(\infty) \ge c\int_0^\infty |\omega(\tau)|\,\mathrm{d}\tau + \int_0^\infty k^4(\tau)\eta^T(\tau)\eta(\tau)\,\mathrm{d}\tau \tag{23}$$

Because the term on the left-hand side is bounded, it follows that $\omega \in \mathcal{L}^1$ and $k^2\eta \in \mathcal{L}^2$.

It is noted that $\dot{k}(t)$, given by Eq. (21), is bounded. (Note that s is bounded because all of its terms are bounded.) Also, $\dot{\omega}$ is bounded because both ω and u are bounded, and $\dot{\eta}$ is bounded because ω, η, and η_0 are bounded. Using the Barbalat's lemma,[12] it follows that

$$\lim_{t\to\infty}\omega(t) = \lim_{t\to\infty}k^2(t)\eta(t) = 0 \tag{24}$$

For a given $\gamma > 0$, from Eqs. (23) and (24), there exist positive functions $c_k(\gamma)$, $c_1(\gamma)$, and $c_2(\gamma)$ such that

$$k(t) < c_k(\gamma), \qquad \forall t, \qquad \int_0^\infty |\omega(\tau)|\,\mathrm{d}\tau < c_1(\gamma)$$

$$\int_0^\infty k^4(\tau)\eta^T(\tau)\eta(\tau)\,\mathrm{d}\tau < c_2(\gamma)$$

The fact that $k^2(t)\eta(t)$ tends to zero does not guarantee that $\eta(t)$ will tend to zero. A question that arises is whether it is possible to assure that $\lim_{t\to\infty}\eta(t) = 0$. This is discussed next.

A. Behavior of $k(t)$

It is first noted that the conditions (24) imply that $\lim_{t\to\infty}s(t) = 0$. [Recall the definition of s given in Eq. (19).] A careful examination of the expression (21) reveals that all three terms in $\dot{k}(t)$ are products of bounded signals and ω or s. Because $\lim_{t\to\infty}\omega(t) = \lim_{t\to\infty}s(t) = 0$ has been demonstrated independently of the behavior of $\eta(t)$ and $k(t)$, it follows that $\lim_{t\to\infty}\dot{k}(t) = 0$. A question that arises is whether in this case it is also possible to show that there exists a condition on gamma > 0 such that $k(t) \ge \bar{k} > 0$ for all time.

Assertion 2: For the adjustment law (21), for any given $k(0) = k_0 > 0$ and $\bar{k} > 0$ where $\bar{k} < k_0$, if there exists a $\gamma > 0$ such that

$$\gamma[c_0(\gamma)c_1(\gamma) + c_2(\gamma)] \le \left(k_0^2 - \bar{k}^2\right)/2 \tag{25}$$

where $c_0(\gamma) = u_m(1+\delta)/\delta + c_k(\gamma)^2$, then $k(t) \ge \bar{k}$ for all $t \ge 0$.

Proof: Because $\omega(t)$ and $s(t)$ are bounded and tend to zero asymptotically, than there exist constants $\bar{\omega}$ and $\bar{s}$ such that $|\omega(t)| \le \bar{\omega}$ and $|s| \le \bar{s}$ for all time. From Eq. (21), we have

$$\dot{k} \ge \frac{\gamma k}{1 + 4\gamma(1-\eta_0)}\left\{u_m\sum_{i=1}^3\left[-\frac{|\omega_i\eta_i|}{|s_i| + k^2\delta} - \frac{|\omega_i|(1+\delta)}{|\omega_i| + k^2(1+\delta)}\right] - |\eta||s|\right\}$$

$$\ge -\gamma k\left\{u_m\sum_{i=1}^3\left[\frac{|\omega_i|}{|s_i| + k^2\delta} + \frac{|\omega_i|(1+\delta)}{|\omega_i| + k^2(1+\delta)}\right] + |s|\right\}$$

Consider the terms within the summation sign,

$$\begin{aligned}\frac{|\omega_i|}{|s_i| + k^2\delta} + \frac{|\omega_i|(1+\delta)}{|\omega_i| + k^2(1+\delta)} &\le \frac{|\omega_i|}{|s_i| + k^2\delta} + \frac{|\omega_i|(1+\delta)}{|s_i| + k^2\delta}\\ &= \frac{|\omega_i|(2+\delta)}{|s_i| + k^2\delta}\\ &= \frac{\left|s_i - k^2\eta_i\right|(2+\delta)}{|s_i| + k^2\delta}\\ &\le \frac{\left(|s_i| + k^2\right)(2+\delta)}{|s_i| + k^2\delta}\\ &= \left(\frac{|s_i|}{|s_i| + k^2\delta} + \frac{k^2}{|s_i| + k^2\delta}\right)(2+\delta)\\ &\le \left(1 + \frac{1}{\delta}\right)(2+\delta)\end{aligned}$$

Hence,

$$\begin{aligned}\dot{k} &\ge -\gamma k\left[3u_m\left(1 + \frac{1}{\delta}\right)(2+\delta) + \bar{s}\right]\\ &= -\gamma k\epsilon\end{aligned}$$

where

$$\epsilon \triangleq 3u_m(1 + 1/\delta)(2+\delta) + \bar{s}$$

Because the lower bound on $k(t)$ is again described by simple first-order dynamics, along the same lines as in the proof of assertion 1 it can be shown that $k(t) \ge 0$ for all time, with $k(t) = 0$ possible only at $t = \infty$.

To prove the aassertion, it is next noted from Eq. (21) that

$$\dot{k} \ge -\gamma k\left\{u_m\sum_{i=1}^3\left[\frac{|\omega_i||\eta_i|}{|s_i| + k^2\delta} + \frac{|\omega_i|(1+\delta)}{|\omega_i| + k^2(1+\delta)}\right] + |\eta^T s|\right\} \tag{26}$$

Because from the Lyapunov analysis it was found that $k(t)$ is bounded, there exists a $c_k > 0$ such that $k(t) \le c_k$ for all time. Also, from Eq. (19) it follows that $|\eta^T s| = |\omega^T\eta + k^2\eta^T\eta| \le |\omega| + k^2\eta^T\eta$.

Hence,

$$\begin{aligned}\dot{k} &\ge -\gamma k\left\{u_m\sum_{i=1}^3\left[\frac{|\omega_i|}{k^2\delta} + \frac{|\omega_i|(1+\delta)}{k^2(1+\delta)}\right] + |\omega| + k^2\eta^T\eta\right\}\\ &= -\frac{\gamma}{k}\left[u_m\sum_{i=1}^3\left(\frac{|\omega_i|}{\delta} + |\omega_i|\right) + k^2|\omega| + k^4\eta^T\eta\right]\\ &\ge -\frac{\gamma}{k}\left\{\left[u_m\left(\frac{1}{\delta} + 1\right) + c_k^2\right]|\omega| + k^4\eta^T\eta\right\}\end{aligned}$$

It follows that

$$k\dot{k} \ge -\gamma\left[c_0(\gamma)|\omega| + k^4\eta^T\eta\right] \tag{27}$$

Upon integrating this equation from 0 to ∞, one obtains

$$k^2(\infty) \ge k_0^2 - 2\gamma\left[c_0\int_0^\infty |\omega(\tau)|\,\mathrm{d}\tau + \int_0^\infty k^4(\tau)\eta^T(\tau)\eta(\tau)\,\mathrm{d}\tau\right] \tag{28}$$

Hence if a γ that satisfies Eq. (25) is found, then $k(t)$ is bounded below by $\bar{k}$ from Eq. (27). ■

Assumming that a sufficiently small $\gamma > 0$ is found that satisfies Eq. (25), then $k(t) \geq \bar{k}$ for all time. In such a case, because $\lim_{t\to\infty} k^2(t)\eta(t) = 0$ and $k(t) \geq \bar{k}$ for all time, it follows that $\lim_{t\to\infty} \eta(t) = 0$.

The properties of the proposed robust dynamic controller are summarized in the following theorem.

Theorem 3: If, under the assumption 2, the control law (18) is used to control the system 1)–3), where $k(t)$ is adjusted using Eq. (21) with a sufficiently small $\gamma > 0$ satisfying the condition (25), then all of the signals in the system are bounded, and, in addition, $\lim_{t\to\infty} \omega(t) = \lim_{t\to\infty} \eta(t) = 0$.

The control law (18) explicitly takes into account the saturation bounds and is robust to bounded disturbances and uncertainty in J.

B. Rate Saturation

The derivative of u_i given by Eq. (18) is of the form

$$\dot{u}_i = -\frac{u_m k \delta(\dot{s}_i k - 2 s_i \dot{k})}{|s_i| + k^2 \delta}$$

Even though the derivative of elements of s is a complicated function of the derivatives of ω, η, and k whose bounds are difficult to calculate, the parameter δ gives some freedom with respect to ensuring that $|\dot{u}_i| \leq \bar{u}_m$, where $\bar{u}_m$ denotes the actuator rate limit. The attractive feature of the proposed control law is that it assures that the control objective is met for any $\delta > 0$. This can aid in finding a δ that assures that the rate limits on the actuators are not violated.

V. Simulations

In this section the properties of the proposed tracking controller will be evaluated through numerical simulations.

A. Inertia Matrix

It is assumed that the inertia matrix of the spacecraft in kilogram-square meter is given by

$$J = \begin{bmatrix} 20 & 2 & 0.9 \\ 2 & 17 & 0.5 \\ 0.9 & 0.5 & 15 \end{bmatrix}$$

and that only an upper bound on the maximum eigenvalue of J (i.e., $\bar{\lambda}_J = 22\ \mathrm{kg}^2\mathrm{m}$), is known to the designer.

B. Disturbances

A square-wave disturbance is added to each axis with periods 40, 50, and 70 s, and magnitudes 0.1, 0.05, and 0.08 Nm, respectively, so that $|z(t)| \leq 0.23$ Nm for all time. It is assumed that the value $\bar{z} = 0.25$ Nm is known to the designer.

C. Noise

It is assumed that the angular velocity measurements are corrupted with random measurement noise of magnitude 0.1 deg per second, and that such a signal is used to compute the control torque. Although, in practice, noisy measurements are first filtered using a suitable filter before being used to calculate the control law, such raw measurements are used here to demonstrate the robustness of the controller.

D. Desired Dynamics

For given desired quaternions ξ and ξ_0, the error quaternions η, η_0 are calculated as

$$\eta = \xi_0 \epsilon - \epsilon_0 \xi + \epsilon^\times \xi$$

$$\eta_0 = \xi_0 \epsilon_0 + \xi^T \epsilon$$

It is recalled that the rotation matrix is given by

$$C = \left(\eta_0^2 - \eta^T \eta\right) I + 2\eta\eta^T - 2\eta_0 \eta^\times$$

and that the angular velocity error is $\omega = \Omega - C\nu$, where ν denotes the desired angular velocity.

E. Controller Parameters

The following values are chosen: $u_m = 0.5$ Nm, $\delta = 0.01$, and $\gamma = 0.01$, whereas the initial condition for the adjustable parameter is chosen as $k(0) = 1$.

F. Simulations

Two simulation cases are considered:

1. Case 1

Point-to-point stabilization, that is, the case when the objective is to regulate $\eta(t)$ to zero from initial conditions $\epsilon(0) = [-0.2\ \ 0.3\ \ -0.4]^T$ to the desired attitude $\xi = [0.1\ \ 0.0\ \ 0.1]^T$ is the first case. In this case $\nu = [0.0\ \ 0.0\ \ 0.0]^T$, and the condition on u_m is $u_m > \bar{z}$, and $u_m - \bar{z} = 0.50 - 0.25 = 0.25$.

2. Case 2

The second case is tracking the desired attitude in the case of time-varying desired angular velocities of the form:

$$\nu(t) = [0.1\cos(t/40) \quad -0.1\sin(t/50) \quad -0.1\cos(t/60)]^T$$

from the same initial conditions in ϵ and ξ. In this case the condition on u_m is $u_m > |\nu|^2 \bar{\lambda}_J + |\dot{\nu}| + \bar{z}$. The maximum values of norms of $\nu(t)$ and $\dot{\nu}$ over an interval of 2000 s are found to be 0.0866 and 0.0061, respectively. Hence $u_m - \|\nu\|^2 \bar{\lambda}_j - \|\dot{\nu}\| - \bar{z} = 0.50 - 0.165 - 0.0061 - 0.25 = 0.08$.

Simulation 1. The response of the closed-loop system in case 1 is shown in Figs. 1–4. It is seen that the control objective is achieved despite the presence of external disturbances

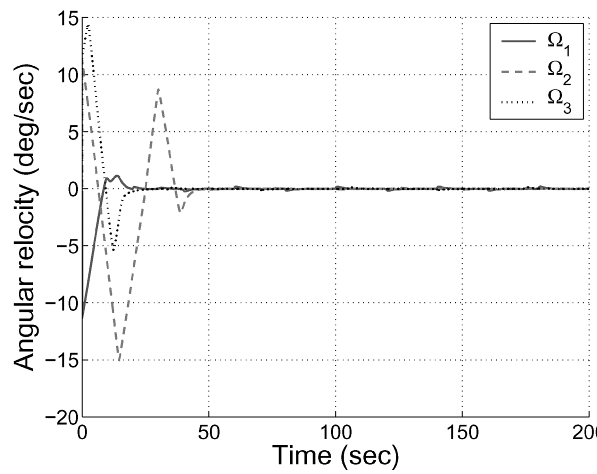

Fig. 1 Case 1: Response of the angular velocities.

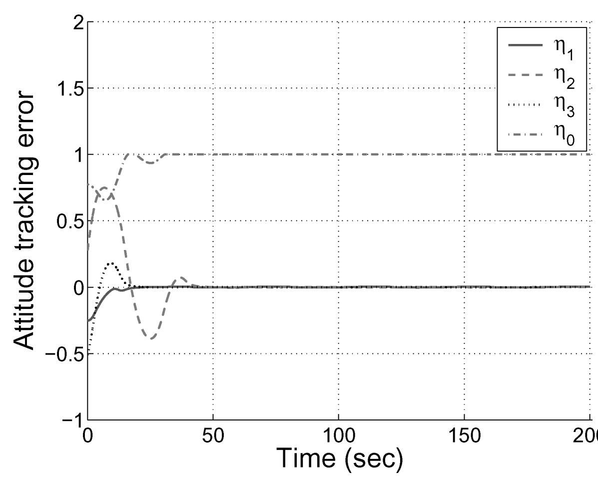

Fig. 2 Case 1: Response of the attitude error.

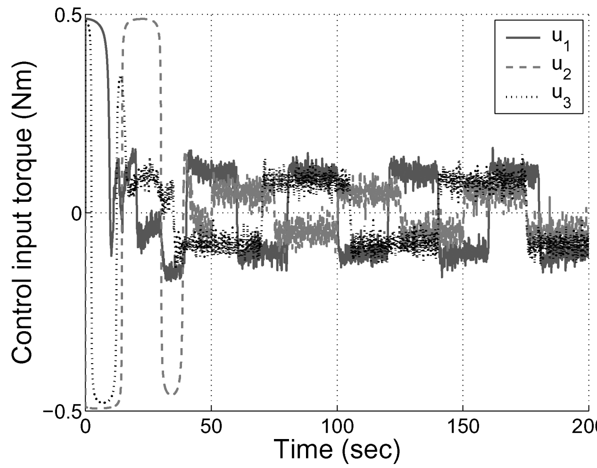

Fig. 3 Case 1: Response of the control input torques.

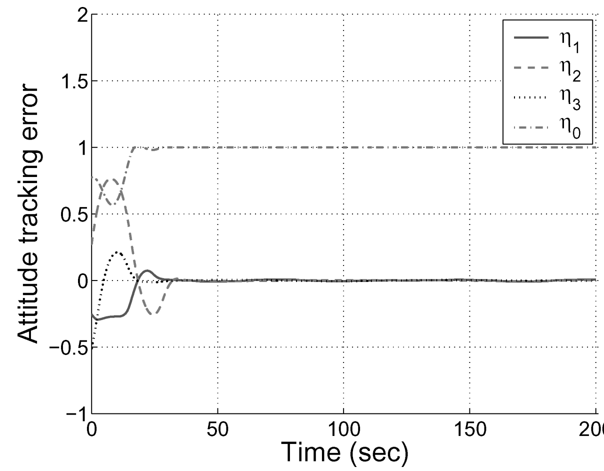

Fig. 6 Case 2: Response of the attitude errors.

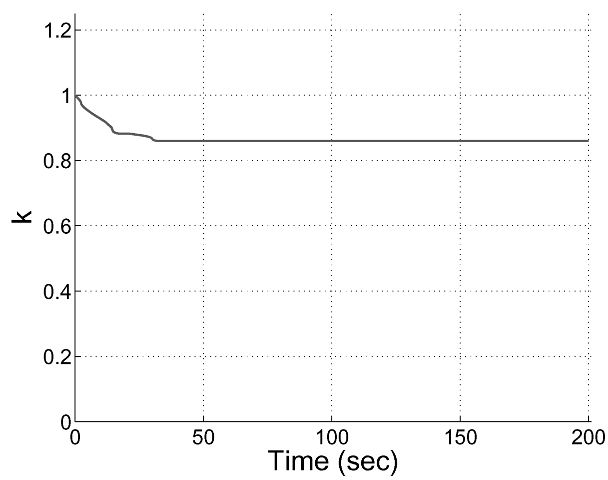

Fig. 4 Case 1: Response of $k(t)$.

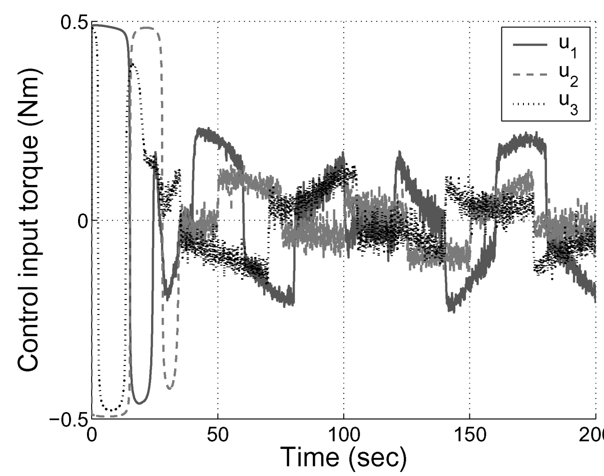

Fig. 7 Case 2: Response of the control input torques.

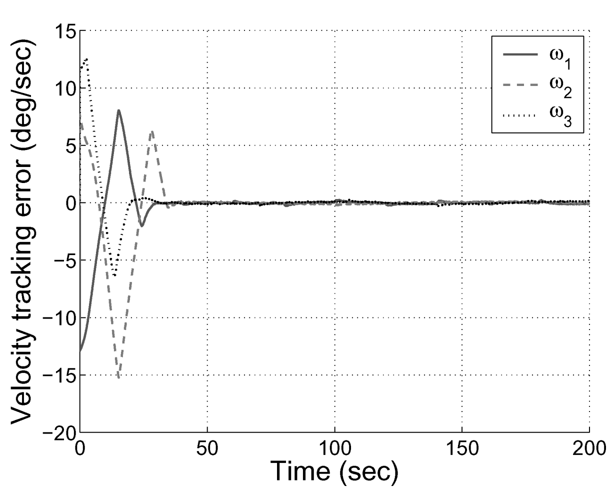

Fig. 5 Case 2: Response of the angular velocity errors.

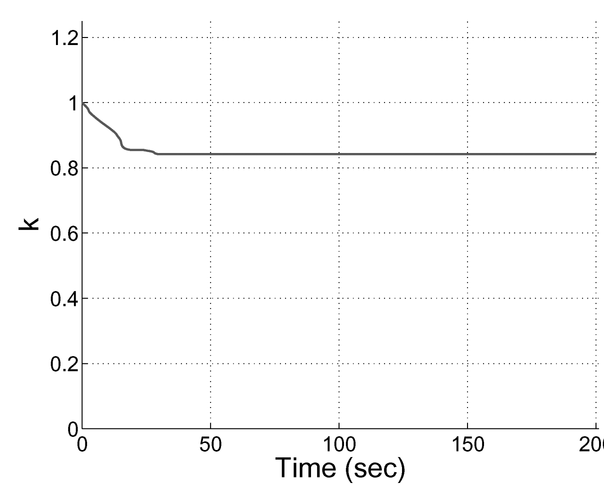

Fig. 8 Case 2: Response of $k(t)$.

and measurement noise. It is also seen in Fig. 4 that $k(t)$ decreases during the transient, but then converges to a constant value.

Simulation 2. The response of the system in case 2 is shown in Figs. 5–8. It is seen that the error quaternions converge to zero (Fig. 6) despite the presence of external disturbances and measurement noise. Figure 8 shows that the response in $k(t)$ is similar to that obtained in the previous cases.

VI. Conclusions

This paper presents a continuous globally stable tracking control algorithm for spacecraft in the presence of control input saturation, parametric uncertainty, and external disturbances. The control algorithm is based on a variable structure control-based design[8,9] and has the following properties: 1) fast and accurate response in the presence of bounded disturbances and parametric uncertainty, 2) explicit accounting for control input saturation, and 3) computational simplicity and straightforward tuning. The paper includes a detailed stability analysis of the resulting closed-loop system. It is shown that global stability of the overall system is guaranteed with continuous control even in the presence of bounded disturbances and parametric uncertainty. In the proposed controller a single parameter is adjusted dynamically in such a fashion that it is possible to prove that both attitude and angular velocity errors will tend to zero asymptotically. The stability proof is based on a Lyapunov analysis and the properties of the quaternion representation of spacecraft dynamics. One of the main features of the proposed design is that it establishes a straightforward relationship between the magnitudes of the available control inputs and those of the desired trajectories and disturbances even with continuous control. Numerical simulations are included to illustrate the spacecraft performance obtained using the proposed controller.

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